

What is "system": some decoherence-theory arguments

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Abstract: We discuss the possibility of making the *initial* definitions of mutually different (possibly interacting, or even entangled) systems in the context of decoherence theory. We point out relativity of the concept of elementary physical system as well as point out complementarity of the different possible divisions of a composite system into "subsystems", thus eventually sharpening the issue of 'what is system'.

1. Introduction. Physical system is described by its parameters (e.g., mass, electric charge etc.) and by the degrees of freedom properly describing dynamics of the system. In the classical world, this general scheme seems inevitable. Nevertheless, in the quantum realm, the things may be different as we show within the context of the "environment-induced superselection rules" (or decoherence) theory.

Actually, the task of dividing complex systems into subsystems is not in general trivial. This fundamental yet a subtle task can be performed in some generality on the basis of the decoherence theory, yet bearing certain open questions. E. g. a composite system \mathcal{C} may not be divisible in respect to the arbitrarily defined "degrees of freedom", thus—relative to *these* degrees of freedom—being an *elementary* physical system (likewise the elementary particles). On the other side, the possible (meaningful) division of \mathcal{C} into subsystems need *not*, in principle, be unique, thus posing the question of physical reality of the "subsystems" emerging from the different possible divisions of \mathcal{C} . Bearing in mind that the real systems are usually open systems, the task of defining "subsystem" coincides with the task of defining "system".

The method employed here is elementary yet conceptually sufficient for addressing the truly fundamental issue of "what is system" within the context of quantum mechanics of complex systems.

2. *The problem.* Most of the "quantum paradoxes" start with the assumption of existence of mutually separable physical systems. On the other side, quantum holism removes most of the problems (except on the intuitive level) from the very beginning—being a consequence of the *fully consistent* quantum mechanical formalism (e.g. of the quantum entanglement). In the macroscopic domain, however, existence of the well-defined, mutually separated systems is the very basis of the physical methods and is actually taken for granted. Thus, in a sense, transferring the concept of the different systems from the macroscopic, through the mesoscopic, to the purely quantum-mechanical domain is at the heart of the problem of the "transition from quantum to classical" [1-3]. This is also a problem of practical importance—since, it seems, that in the *realistic* situations, we are able to distinguish between the different systems (e.g. between the object of measurement and the measurement instrument).

The possibility of defining mutually independent (compatible) degrees of freedom (and their conjugate momenta) and of performing independent ("local") measurements of the observables is a *defining feature* of the different physical systems. The importance of the issue is rather apparent. E.g. according to Zurek [2]: "...[quantum mechanical] problems... cannot be even posed when we refuse to acknowledge the division of the Universe into separate entities", while "...without the assumption of a preexisting division of the Universe into individual systems the requirement that they have a right to their own states cannot be even formulated". Of course, the rules for defining the preferred states (e.g. the pointer basis, as well as the pointer observable) of an open system comes from the foundations of the decoherence theory. On this basis appeared an early draft [4] of the problem considered here.

This issue should be distinguished from the problem of the loss of individuality of mutually *entangled* systems. Actually, the entangled states refer to the, initially, well-defined systems: the systems (actually subsystems) are usually assumed already to be *defined*, as well as their state spaces, which is the basis for defining the entangled states. So, in this perspective, the task of answering 'what is system' is a more fundamental task than investigation of entanglement itself.

Essentially the same problem has recently been addressed e.g. by Zanardi et al [5] and by Barnum et al [6] (and the references therein), by considering the different possible operational uses of entanglement in the quantum information issues. While bearing some similarity with our results, the results therein presented are based on the different approaches that is briefly

discussed in Section 5. Here, we employ the foundations of the decoherence theory and particularly certain recent results in this regard [4, 7, 8].

3. Separability. As implicit in the above quotation of Zurek, the initial definitions of the subsystems *make sense if one can a posteriori justify these definitions* on the basis of the *occurrence of decoherence*. The relevance of this statement seems to be apparent in the macroscopic, not necessarily yet in the mesoscopic or microscopic (fully quantum mechanical) context.

A detailed analysis of the occurrence of decoherence points out the condition of *separability* (cf. DEF. 1 below) of the interaction term of the Hamiltonian as the (effective) necessary condition for the occurrence of decoherence. Investigating the occurrence of decoherence is truly a subtle task [7-9]. E.g., separability of the complete Hamiltonian (of the composite system "system + environment") is sufficient in this regard [7, 8]. Strong interaction allows the occurrence of decoherence, which still depends on a number of the details in the model of the system [7]. On the other side, strong interaction is not necessary for the occurrence of decoherence [9]. Nevertheless, the condition of separability in the 'macroscopic context' of the theory represents an (effective) necessary condition for the occurrence of decoherence [8].

Now, the *separability appears as a condition useful for defining the 'dividing line'* between the subsystems. Formally, existence of the subsystems is presented (cf. (1) below) by the tensor-product symbol, \otimes , while assuming the definitions of the subsystems through their—implicitly present—degrees of freedom.

DEF. 1: A bipartite $(1 + 2)$ system's observable \hat{A}_{12} is of the *separable kind*, if its general form

$$\hat{A}_{12} = \sum_i \hat{B}_{1i} \otimes \hat{C}_{2i}, \quad (1)$$

fulfills any of the following, mutually equivalent conditions: (A) Its spectral form reads $\sum_{i,j} a_{ij} \hat{P}_{1i} \otimes \hat{P}_{2j}$, where appear the (orthogonal) projectors onto the Hilbert spaces of the two systems; (B) there exist the two orthonormal bases in the state spaces of the systems, $\{|i\rangle_1\}$, and $\{|\alpha\rangle_2\}$ that diagonalize the observable: ${}_1\langle i|\hat{A}_{12}|j\rangle_1 = 0, \forall i \neq j$, and ${}_2\langle \alpha|\hat{A}_{12}|\beta\rangle_2 = 0, \forall \alpha \neq \beta$; (C) every pair of the observable of the system 1 in (1) mutually commute, and analogously for the system 2.

A constructive proof of existence of the general form (1) of a bipartite system's observable is given in [8]. Depending on the actual task, any of these

definitions of separability may equally be operationally useful. Needless to say, a bipartite system's Hamiltonian is subject to DEF. 1.

Therefore, operationally, investigating separability of the Hamiltonian gives rise to both [7, 8]: (i) to the superselection rules defined by the projectors $\{\hat{P}_{1i}\}$ (when the system 1 is considered as the open system), and (ii) to a definition of the pointer observable and therefore of the possible pointer basis (or of the "preferred set of states") of the open system—e.g. the system 1 in our notation). Having in mind that the observables, e.g. \hat{B}_{1s} , are the functions of the degrees of freedom of the system 1, the task of investigating decoherence actually assumes the *initially* well-defined (sub)systems.

Bearing in mind the subtleties concerning the occurrence of decoherence, we simplify our approach: actually, we employ the condition of separability as a criterion for making the dividing line between the subsystems of a composite system.

4. *Quantum relativity of "system"*. Usefulness of separability in the foundations of decoherence theory bears some subtlety yet. The example of the hydrogen atom is paradigmatic in the following sense. The composite system "hydrogen atom (HA)" is originally defined by the Hamiltonian:

$$\hat{H} = \hat{T}_e \otimes \hat{I}_p + \hat{I}_e \otimes \hat{T}_p + \hat{V}_{Coul}, \quad (2)$$

where the Coulomb interaction couples the positions of the electron (subscript e) and the proton (subscript p), bearing obvious notation. Having in mind the definition of separability (Section 3), it is straightforward to prove non-separability of \hat{H} yet separability of the Coulomb interaction¹.

However, the proper *canonical transformations* of the degrees of freedom give another, *separable form* of \hat{H} ; even more, each single term is (apparently) of the separable kind:

$$\hat{H} = \hat{T}_{CM} \otimes \hat{I}_R + \hat{I}_{CM} \otimes \hat{T}_R + \hat{I}_{CM} \otimes V_{Coul}(\hat{r}_R), \quad (3)$$

where CM stands for the "center of mass" and R for the "relative particle" system; $r_R \equiv |\vec{r}_e - \vec{r}_p|$.

In the *context* of our considerations, these well-known transformations give rise to the following observation. The composite system HA is decom-

¹The *strength* of the separable interaction term gives rise to the bound states and the interpretation of HA as distinguished in the body text.

possible² into the pair of the quantum particles (e, p) , stemming from separability of the *strong* Coulomb interaction (cf. footnote 1). On the other side, the form (3) of the Hamiltonian refers to the new, also well known, division of HA: the system now reads "Center of mass + relative particle" ($CM + R$); certainly, $e + p = \mathcal{C} = CM + R$. Due to the small mass-ratio $m_e/m_p \ll 1$ —it is (semiclassically) allowed to "identify" CM with p and R with e . Nevertheless, in general, this identification is not physically reasonable, as we show in the sequel. From this example, we learn:

the choice of the degrees of freedom may redefine the Hamiltonian separability, thus (cf. Section 3) directly referring to the issue of putting the dividing line between the (sub)systems.

Let us first briefly consider the case of totally nonseparable Hamiltonian. That is, we assume that a given Hamiltonian can not be (re)written in a separable form by the use of any (linear) canonical transformations. As to the told in Sections 2 and 3, then one can not define a dividing line between the "subsystems" of the composite system defined by the Hamiltonian considered. Then, it seems we are forced to consider the system *undivisible*, thus resembling the concept of elementarity of the quantum particles. Physically, a definition of the subsystems in this case is artificial, and the measurements of the "subsystems' observables" is nothing but the measurements of the observables of the composite system, not yet interpretable in terms of the observables of the well-defined subsystems.

As a counterexample, let us analyse the following possibility. A Hamiltonian is separable *relative* to a set of the "degrees of freedom" (and their conjugate momenta), $(\hat{x}_{Ai}, \hat{p}_{Aj}; \hat{\xi}_{Bm}, \hat{\pi}_{Bn})$, thus defining a division of the composite system as $\mathcal{C} = \mathcal{A} + \mathcal{B}$; by definition, $[\hat{x}_{Ai}, \hat{p}_{Aj}] = i\hbar\delta_{ij}$ (and analogously for \mathcal{B}), while $[\hat{x}_{Ai}, \hat{\xi}_{Bm}] = 0$ and $[\hat{x}_{Ai}, \hat{\pi}_{Bn}] = 0$ (and analogously for \hat{p}_{As}). But, suppose that the same Hamiltonian can be rewritten in a separable form relative to another (analogous) set of the "degrees of freedom", $(\hat{X}_{Dp}, \hat{P}_{Dq}; \hat{\xi}_{E\alpha}, \hat{\Pi}_{E\beta})$, thus giving rise to another possible division of the composite system, $\mathcal{C} = \mathcal{D} + \mathcal{E}$. By the assumption: the two sets of the degrees of freedom are mutually related by the linear canonical transformations

$$\hat{\xi}_{E\alpha} = f_{\alpha}(\hat{x}_{Ai}, \hat{p}_{Aj}; \hat{\xi}_{Bm}, \hat{\pi}_{Bn}), \quad \hat{\Pi}_{E\beta} = g_{\beta}(\hat{x}_{Ai}, \hat{p}_{Aj}; \hat{\xi}_{Bm}, \hat{\pi}_{Bn}), \quad (4)$$

²Here, we do *not* assume the occurrence of decoherence in HA—the proton is much too small in order to play the role of the environment for the electron. We just formally employ the separability of the Coulomb interaction to point out consistency of our considerations.

and analogously for the subsystem \mathcal{D} , while assuming the inverse is also defined. Needless to say, the measurements of e.g. \hat{x}_{Ai} , or $\hat{\zeta}_{Dp}$, may be interpreted as the measurements of the observables of the composite system. Yet, the supposed separabilities of the Hamiltonian allow the interpretations in terms of the subsystems, again bearing some subtlety.

In general, the measurements of e.g. the observables of \mathcal{A} partly reveal, yet *quantum mechanically undetermined* values of the observables of both \mathcal{D} and \mathcal{E} —due to (4), one may obtain e.g. $[\hat{x}_{Ai}, \hat{\zeta}_{D\alpha}] \neq 0$. As a consequence: the inverse of (4) can not be used for determining the definite values of the observables of \mathcal{D} and \mathcal{E} —in contradistinction with the macroscopic experience. On the other side, only the measurements of \mathcal{A} and \mathcal{B} (of \mathcal{D} and \mathcal{E}) do not mutually interfere, referring to the mutually compatible observables. Therefore, the measurements of the observables of the "subsystems" belonging to the different divisions do not make sense, while the measurements referring to the observables of the subsystems belonging to the *same division* of the composite system are physically reasonable. Needless to say, the later is in agreement with the standard, general procedure we have learnt in the "classical domain" (Section 2—cf. also Section 5). As a consequence, the two possible divisions may refer to the two, mutually complementary, possible entanglements in the system \mathcal{C} : $\sum_i c_i |\psi_i\rangle_A \otimes |\chi_i\rangle_B$, and $\sum_j d_j |\psi_j\rangle_D \otimes |\phi_j\rangle_E$ (compare to [5, 6]).

As long as the composite system may be considered to be *isolated*, the two different divisions as described above seem mutually equivalent for an independent observer. This, however, need not be the case for an open composite system, as discussed in [10].

It is probably obvious: a definition of e.g. subsystem \mathcal{A} makes sense *if and only if* the subsystem \mathcal{B} is *simultaneously* defined. This is both a mathematical consequence of the canonical transformations as well as physically a reasonable notion.

Therefore, the concept of *elementarity* as well as of a *subsystem* are *relative*; as to the later, bearing in mind that the real systems are usually open, the relativity of "subsystem" actually means relativity of the basic physical concept of "system".

5. Discussion. Depending on the context, we hope the use of the term "separability" is clear—since it applies to both, the total Hamiltonian as well as to any (e.g. interaction) term of the Hamiltonian. Since this is not substantial, we do not explicitly distinguish between these possibilities

throughout the paper. So, for simplicity, we assume that '(non)separability' (non)defines the 'dividing line'—which is the main operational tool of our analysis.

Originally, the problem of "what is system" (Section 2) stems from the "macroscopic context" of the decoherence theory [1-3]. It seems that, nowadays, the physicists are ready to accept the "undivided universe" on the truly microscopic, and partly on the "mesoscopic" scale. Yet, in the context of the "macroscopic considerations", it seems unplausible to start with such a hypothesis [1, 3]. Therefore, our conclusions mainly refer to the macroscopic context of the decoherence theory; investigating their extension towards the fully quantum mechanical scales remains an open task of our analysis. To this end, the above distinguished use of the condition of separability may be employed as a (plausible) working hypothesis.

Following the fundamentals of the decoherence theory (yet mainly in the macroscopic context), we have argued that the condition of separability of the Hamiltonian may serve as a criterion for making a "division line" between the subsystems of a composite system. As we here emphasize, this reasonable approach gives automatically rise to the possibility of defining the subsystems, through a *definition* of the degrees of freedom (and their conjugate momenta) that are *based on the condition of separability* of the Hamiltonian. We have also seen that the condition of separability is consistent with our macroscopic experience: e.g. the measurements on \mathcal{B} (\mathcal{A}) may be performed *independently* on the measurements on the subsystem \mathcal{A} (\mathcal{B}); as a benefit, the subsystem \mathcal{B} (\mathcal{A}) may be defined *only simultaneously* with the subsystem \mathcal{A} (\mathcal{B}). The different divisions of the composite system may bear quantum mechanical complementarity, being mutually exclusive divisions of a composite system. This way, the problem of "what is system" seems to be sharpened, and particularly *reduced* to the following problem: "as to what extent, one may ascribe the physical reality to the different divisions of a composite system, especially in the 'macroscopic' context". Needless to say, much remains yet to be done in this respect. To this end, the example of the hydrogen atom is useful both, as an example of applicability of our general models to the *realistic systems* as well as an example exhibiting the subtleties in the same concern. As to the later, the strength and separability of the Coulomb interactions relative to the degrees of freedom of the pair (e, p) justify the division of the atom into the pair "electron + proton" (cf. footnotes 1 and 2). Nevertheless, in general, identification of the subsystems referring to the different divisions of the composite system (semi-classically plausible

identification: $CM = p$, and $R = e$ in HA) are physically nonjustifiable—complementarity of the different divisions of a composite system.

Once made, a division of a composite system \mathcal{C} can be straightforwardly extended (further "coarse graining" of \mathcal{C}) in accordance with the above criteria—e.g. $\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2 + \dots$

Finally, our discussion and conclusions are applicable virtually to any complex quantum system. However, its relevance to the realistic systems remains yet to be investigated in the purely quantum-mechanical ("microscopic") as well as in the mesoscopic context. To this end, it is interesting to compare our approach and conclusions with the approach of Zanardi et al [5] and Barnum et al [6]. A common element of [5] and our considerations is the notion on the importance of interaction in the composite system in defining the subsystems. While this is a conclusion in [5], we still use this plausible notion (stemming from the decoherence theory) in developing the general models of our analysis. The approach of Zanardi et al [5] is based on certain axioms referring to the "experimentally accessible observables", which is yet an open issue of our approach. Our approach is characterized by the pointing out separability as an operational tool in defining the subsystems, yet bearing possibly some restrictions onto the "macroscopic context". On the other side, rejecting the "reference to a preferred subsystem decomposition" of a composite system, Barnum et al [6] seem essentially to point to the relativity of the concept of "subsystem"—in analogy with our conclusion. However, being an operational analysis of entanglement, their paper does not directly tackle the issue of "what is system". Nevertheless, we believe, that the conclusions of [5, 6] are consistent with our conclusions, which still follow from the foundations of the decoherence theory.

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